Multi-hop Energy Sharing in Rechargeable Wireless Sensor Networks

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Abstract: The emerging energy-sharing technique is an alternative way to address the energy-limited problem in Wireless Sensor Networks (WSNs). This paper argues that nodes transfer energy by a novel manner, *multi-hop energy sharing*, by which, a multi-hop network can realize self-organized energy delivering among nodes instead of using additional vehicles, such as mobile charger. There may exist several possible energy sharing paths between each pair of nodes, and not all of them are feasible because of the inherent physical properties during energy sharing. This paper develops ways to find those feasible paths. By the energy-sharing technique, this paper proposes a Multi-hop Energy Sharing Scheme (MESS) to find feasible node pairs so that the overall network performance can be maximized. A metric *reward* is applied to measure the performance improvement. MESS considers two energy sharing cases: static and dynamic, according to the factors affecting the remainder energy of each node. Two algorithms are correspondingly designed: Static Energy Sharing Algorithm (SESA) and Dynamic Energy Sharing Algorithm (DESA). Theoretical analysis proves that the overall reward achieved by both algorithms, SESA and DESA, are all $1 - 1/e$ of that by the optimal one, and the energy consumption of the networks using these two algorithms during energy sharing is also bounded. In the dynamic case, the reward obtained by DESA has an additional error with the expectation of $\mathbb{E}(\Delta\tau)$, where $\Delta\tau$ is the reward difference between the reward obtained by DESA and that by the optimal one at each time slot τ. This paper also conducts detailed simulation to evaluate our scheme. The simulation results show that MESS can greatly improve the fairness of the energy consumption among the whole network by consuming a relative small amount of energy.

Keywords: Multi-hop Energy Sharing; Wireless Sensor Networks; Energy Harvesting.

1 Introduction

In recent years, a new technique to prolong network life is increasingly researched, which is energy sharing. To achieve the long-term operation of WSNs, classical methods to alleviate the energy limitation can be roughly classified into two groups: energy conservation and extra energy-supplement. The former is mainly realized by designing energy-ware protocols or platforms/hardware, while the later refers to replacing batteries or harvesting natural sources by additional modules, such as solar panel [1] and wind or vibration energy generators. The energy harvesting technique can support a network to operate permanently with the extra energy supplement. But some environment factors, such as the shadow of clouds, cause that nodes have different and time variable harvested energy profiles. So energy harvesting can worsen the energy unbalance among nodes. Another method is to charge the low-energy nodes by wireless charge vehicle [2]. WSNs may be deployed in wild applications [3][4][5] and it is either expensive to replace batteries or difficult to charge nodes by mobile charger. In both of the previous group ways, the serious problem is the energy unbalance among nodes, which results in low energy efficiency and short network life [6]. This paper is interested in the new technique, *energy sharing*, which can be an alternative way to alleviate the energy unbalance in network, especially when nodes can harvest natural sources [7]. In WSNs, energy sharing technique has been increasingly investigated and applied in recent years [7][8][9][10][2], because it can be beneficial for some applications, such as wearable computing [11], green building [12] and so on. Most of existing researches on energy sharing transferred energy by mobile charger and attempted to find optimal paths for it. These energy sharing methods are actually peer-to-peer and have no essential difference with that in Radio-Frequency Identification (RFID) systems [13]. During the process of energy charge in RFID systems, the potential energy receiver must be passive RFID tags while the energy source must be readers, which have rich energy [14]. Different from the previous energy sharing schemes, this paper argues a novel way, *multi-hop energy sharing*, to share energy among nodes by excluding the mobile chargers.

Multi-hop Energy Sharing. The novel scheme, multi-hop energy sharing, has the inherent property of the multi-hop energy delivering, different from previous schemes. The basic idea behind the multihop energy sharing is that energy can be treated like data, and hence can be delivered among network like data packet [7][15]. But the multihop energy sharing is different from data delivering on some aspects because of the physical properties during energy sharing. Firstly, the interference model for the multihop energy sharing is quite different from the wireless communication interference model. For example, the interference range is modeled as circle or analogue in most interference models for wireless communication while the electromagnetic coverage is totally different during wireless energy charge. In wireless communication, receiver cannot receive data when it is interfered. Interestingly, the nearby receivers may be charged some unexpected energy because of the energy dissipation caused by the energy sharing among other nodes. Secondly, energy must flow from nodes with higher energy level to those with lower one. It is the inherent physical property during energy charging. During data delivering, the fact that a node can be a receiver or transmitter is not impacted by its energy level only if it has enough energy to receive or transmit data. Thirdly, differing from the energy charge in the RFID systems with passive tags, where the possible energy receiver must be RFID tags and energy source be readers with rich energy source [14][16], multihop energy sharing enables any node possible to be a source or a target node or able to relay energy for other nodes.

These characters indicate that the protocols for data delivering cannot be applied for the multi-hop energy sharing though Zhu *et al.* argued that energy can be transmitted like data [7]. Furthermore, in energy harvesting WSNs, nodes able to harvest more source than other nodes, should afford of more tasks or share their energy with others. Otherwise, the harvested energy must be over accumulated so as to be wasted because of the limited capacity of nodes' batteries. Zhu *et al.* designed a hardware platform composed of energy routers and related energy access and networking protocols to route energy efficiently and quantitatively among embedded sensor devices [7]. This device proved that it is feasible to share energy among nodes by multi-hop mode. The above facts motivate us to develop a novel multi-hop energy sharing scheme, and to design algorithms to implement it.

Contribution of this paper. Based on the increasingly researched and applied technique, energy sharing, this paper develops a novel *Multi-hop Energy Sharing Scheme* ($MESS$) so as to share energy among nodes in multi-hop mode. To our best knowledge, this paper is the first to analyze the energy sharing in multihop mode. Conditions for the feasible mulithop energy sharing paths are developed firstly in this paper. Instead of designing scheme to look for the multihop energy sharing paths directly, this paper designs another scheme relatively more easy to implement, called MESS. In MESS, this paper respectively designs algorithms: $SESA$ and $DESA$, respectively for two cases: static and dynamic energy sharing. In the first case, during energy sharing, there is no other actions taken to consume energy. Thus, the remainder energy of each node is *static*, and its changing is caused only by energy sharing. For example, nodes cannot harvest solar energy during night time and each node implements no task except the energy sharing. In the second case, the remainder energy of each node is *dynamic*. It may be variable because of some factors including the energy harvesting, task implementation and impact of other nodes' energy sharing. For these two cases, our algorithms consider two opposite sides of energy sharing: the reward and the cost to share energy. Our algorithms are nearly optimal on maximizing the network reward while its energy cost is bounded. The contributions of this paper are as follows.

- An almost optimal scheme $MESS$ is designed for the multi-hop energy sharing in WSNs. $MESS$ considers two cases: static and dynamic, and composes of two algorithms: $SESA$ and $DESA$ respectively designed for these two cases. Our theoretical analysis shows that the overall reward is at least $1 - 1/e$ of the optimal scheme by both $SESA$ and $DESA$. In the dynamic case, the expected error of the reward obtained by $DESA$ can be $\mathbb{E}(\Delta\tau)$, where $\Delta\tau$. The energy cost because of the energy sharing is also bounded by both of two algorithms in $MESS$.
- Simulation evaluations: this paper conducts simulations to evaluate our algorithm based on the OMNeT++ simulation platform [17], and analyzes the performance of the two algorithms in large-scale networks.

Road map. The rest of this paper firstly defines the power dissipation model and gives the reward function in Section 2. The feasible condition and the outline of our solution for multi-hop energy sharing are given in Section 3. Our scheme for multi-hop energy sharing is presented and analyzed in Section 4. The simulation is established and its results are analyzed in Section 5. Section 6 surveys the related works. The whole paper is concluded in Section 7 and some future works are also discussed.

2 System Model

This section states the network, power-dissipation and reward models.

Suppose that there are n sensor nodes composing a set $V = \{v_1, v_2, \cdots, v_n\}$, forming a network. The node density of this network must be high enough to ensure each node able to transfer its energy to at least one neighbor. Each node is equipped with modules able to (1) harvest natural sources from environment, (2) share energy with others. Assume that the module to share energy can adjust its angle to proper direction so it can transfer energy to its receiver efficiently. Each node also has capacity-limited component, such as rechargeable battery or capacitance, to store energy. The maximal capacity of each node is denoted by B. There is always the constraint $0 \le E_i^m(\tau) \le B$ for each node, where $E_i^m(\tau)$ denotes the remainder energy of node v_i at the beginning of time slot τ. $E_i^m(\tau)$ contains several parts: the remainder, consumed, harvested, transferred and received energy at the previous time slot, respectively denoted by $E_i^m(\tau-1)$, $E_i^c(\tau-1)$, $E_i^h(\tau-1)$, $E_i^t(\tau-1)$ and $E_i^r(\tau-1)$. Here, $E_i^t(\tau-1)$ denotes the energy that v_i transmits to others and includes the energy consumed on the way to target node, *i.e.*, $E_i^c(\tau)$. Actually, the consumed energy is quite small comparing to the capacity of the energy storage device, such as AA type battery. Most of symbols in this paper and their meanings are given in Table 1.

Table 1 Notations and symbols

Symbol	Meaning
f	The reward function
E	Energy
v_i	Node ID
θ_{ij}	Energy sharing action from v_i to v_j
τ	Time slot
S	Action set
c	Energy sharing cost
$\mathcal{X}$	Set of source nodes
$\mathcal{R}$	Set of target nodes
k	The cardinality of $\mathcal{X}$
$\mathcal{N}_i$	Neighborhood in v_i's maximal energy-sharing range

In the process of energy sharing, some energy is inevitably consumed and dissipated according to the research results in [18]. Although the power consumption caused by energy sharing relates to several parameters, it is mainly determined by the distance between the source and its target because most parameters of an energy transceiver device are fixed. Let d_{ij} denote the distance between nodes v_i and v_j. When node v_i transfers energy to v_j, the energy consumption on distance d_{ij} is $c_{ij} = \alpha d_{ij}^{\beta}$, where α and β are positive constants. Let d_{max} denote the maximal distance in which a node v_i can transfer its energy to its one-hop neighborhood so it has the maximal neighborhood $\mathcal{N}_i$. During energy sharing, energy dissipation is the physical phenomenon and may cause the main energy lost in wireless energy sharing. Energy dissipation also causes nodes to transmit some energy to unexpected destination. For example, node v_1 dissipates parts of its energy to node v_5 when it is transferring energy to node v_2 in Figure 1. This paper does not require any knowledge about the distribution models of the energy harvesting and dissipation. Meanwhile, each node is also

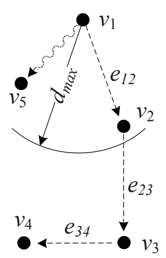

Figure 1 Multi-hop energy sharing. There is an energy sharing path from v_1 to v_4.

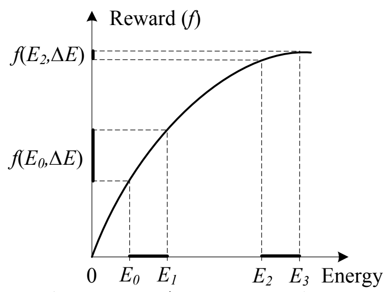

Figure 2 A convex and monotone reward function

able to harvest energy. According to the related works given in Section 6, the amount of harvested energy at each time slot, such as an hour, is also much smaller than the capacity of the energy storage device.

This paper introduces the concept: *energy sharing action*[a], and denotes it by θ. Taking an action means that a transmitter transmits some energy to a receiver. An reward function is also introduced to measure the reward returned to an action. The reward f of each action is the function of the receiver's remainder energy E^r and the shared energy ΔE, and calls this function as reward function, *i.e.*, $f(E^r, \Delta E)$. When an action θ_{ij} is taken, *i.e.*, v_j receives some amount ΔE of energy from node v_i, the action obtains some reward, denoted by $f(\theta_{ij})$. Notice that two nodes v_i and v_j are neighboring in the action θ_{ij}. Thus, we have $f(\theta_{ij}) = f(E_j^r, \Delta E)$. The formula of the reward function depends on the applications, and previous works designed different functions to describe the reward that a network can obtain in sensor activation [19], coverage [20][21] and so on. Many previous works assumed the reward function is non-decreasing, monotonic and convex. Intuitively, nodes affording of more task remain less energy comparing to other nodes. Thus, it needs energy more eagerly than others. In this paper, the reward function f_i of receiver v_i is assumed to be monotone and convex as shown in Figure 2. Both the remainder energy of the receiver and the shared energy have impact on the reward of an action. For example, two actions are taken respectively at two moments, when the remainder energy of v_j is E_0 and E_2. Although the amount of received energy by v_j equals to ΔE in both cases, where $\Delta E = E_1 - E_0 = E_3 - E_2$, obviously $f(E_0, \Delta E) > f(E_2, \Delta E)$ because $E_0 < E_2$ and v_j thirsts after more energy at the point of E_0 than at the point of E_2. This paper also assumes that the reward function is non-decreasing, monotone and convex.

Notice that an action θ_{ij} consumes the transmitter v_i some energy, which including two parts: some received by its receiver v_j and other dissipating on the way from v_i to v_j. The former part is returned from its target

[a]In the following context, *energy sharing action* is shorten to *action*

nodes with some reward while the later is wasted on the way of energy sharing. We call the later part as the cost of the action θ_{ij}. When an energy sharing scheme is adopted, we always hope the scheme costs as few energy as possible. Hence, the cost of the scheme is another important metric to evaluate its performance.

In a multi-hop network, the overall reward function is denoted by f_V, *i.e.*, $f_V = \sum_{v_j \in \mathcal{R}} f_j$, where f_i is the reward function for node v_i, $i = 1, 2, \cdots, n$. Assume f_V to be symmetric in the rest of this paper, which is also non-decreasing, monotonic and convex as the assumption given previously. In the following context of this paper, S denotes a set of actions, and the total reward of the set of actions is denoted by $f(S)$, *i.e.*, $f(S) = \sum f_{\theta \in S}(\theta)$.

3 Problem Formulation and Feasible Solution

3.1 Problem Formulation

This block formalizes the energy sharing problem as follows. At each time slot $\tau \in T$, a set of source nodes is selected. These source nodes find their target nodes respectively, which may be mulithop away. Each source node v_i transmits its energy to its target node through the relay nodes hop by hop. There then forms an energy sharing path between each source node and its target node. This means that each energy sharing path composes of several actions. In each time slot τ, there are several actions finished. The transmitter and receiver[b] nodes of these actions form two sets $\mathcal{X}_\tau$ and $\mathcal{R}_\tau$. During this process, each action $\theta_{ij}(\tau)$ is taken so as to maximize its reward $f_j(\theta_{ij}(\tau))$. Because there may be more than one source nodes, a series of actions $\theta_{ij}(\tau)$, $v_i, v_j \in V$ and $i \neq j$, are selected out at each time slot τ, and forms an action set S_τ. The overall reward obtained by all actions at this time slot is denoted by $f(S_\tau)$, where $f(S_\tau) = \sum_{\theta_{ij}(\tau) \in S_\tau} f_j(\theta_{ij}(\tau))$. Furthermore, a series of action sets S_τ, $\tau \in T$, are selected in a period T. The total reward obtained by this series of action sets in T is given as $\sum_{\tau \in T} f(S_\tau) = \sum_{\tau \in T} \sum_{\theta_{ij}(\tau) \in S_\tau} f_j(\theta_{ij}(\tau))$.

When an action is taken $\theta_{ij}(\tau)$, some energy c_{ij} is correspondingly consumed. The overall cost of all actions in the period is $\sum_{\tau \in T} \sum_{\theta_{ij}(\tau) \in S_\tau} c_{ij}$. Thus, an optimal multi-hop energy sharing scheme is actually to find a series of action sets so that the overall reward $f_V(T)$ is maximized, as Equation (1), while its overall cost is bounded in each period.

This paper formulates the energy sharing problem as a convex optimization problem with the object to maximize the overall network reward as follows:

$$\max \sum_{\tau \in T} \sum_{\theta_{ij}(\tau) \in S_\tau} f_j(\theta_{ij}(\tau)) \tag{1}$$

$$s.t. \quad 0 < f_j(\theta_{ij}) < f_j(E_i^t(\tau - 1)), \forall v_i \in \mathcal{X}_\tau, v_j \in \mathcal{R}_\tau, \forall \tau \tag{2}$$

$$\begin{aligned} E_i^m(\tau) &= E_i^m(\tau - 1) + E_i^h(\tau - 1) + E_i^r(\tau - 1) \\ &\quad - E_i^t(\tau - 1) - E_i^c(\tau - 1), \forall v_i, \forall \tau \end{aligned} \tag{3}$$

$$E_i^m(\tau) \leq E_{max}^m, \forall v_i, \forall \tau$$

The first inequality in Equation (2) can hold because the amount of energy the receiver v_j received is less than that its transmitter transferred, and each action must achieve positive reward. Equation (3) is an energy updating function for each node. In Equation (3), $E_i^r(\tau) = 0$ when node v_i is a transmitter, and $E_i^t(\tau) = 0$ when node v_i is a receiver. Each transmitter must consume its own energy $E_i^t - E_j^r$ because of energy sharing, where v_i is a transmitter and v_j is its receiver. E_i^c is energy consumption of v_i because of another factors except energy sharing. Unfortunately, it is NP-hard to find the optimal scheme and the proof for its hardness is similar to that for Theorem 3.1 in [20]. This section formulates the multi-hop energy sharing problem and gives ways to find its feasible solutions.

3.2 Feasible Multi-hop Energy Sharing Path

The solution to the problem (1) is actually to find source nodes and their target nodes, and also the energy sharing paths among them so that the overall reward can be maximized. However, the energy transmission is different from data transmission because the former requires that energy should be transmitted from nodes with higher energy level to those with lower energy level. Precious methods on data transmission cannot be applied directly to this paper. This section gives out the outline of our solution to the problem (1). It is quite challenging to find optimal solution to the problem. This section gives the way to find the feasible solution. We firstly highlight what is the feasible solution as the following definition.

Definition 3.1: A multi-hop energy sharing path is feasible if there is no any relay node which must block this multi-hop energy sharing, and the overall reward obtained by it is positive.

In the whole network, there may be several simultaneous multi-hop energy sharing paths. Several source nodes may transmit energy to their source nodes in parallel so there may be several actions taken each time. The example in Figure 3 shows how the process of a multi-hop energy sharing works, and then clarify which kind of multi-hop energy sharing paths are feasible. In this example, another concept “round” is introduced. Actually, a round is the collection of actions taken at time slot τ. These actions may be taken for different paths. There two paths from v_1 to v_4 and from v_k to

[b] In this paper, transmitter and receiver are nodes transmitting and receiving energy and related to the concept: action. The source and target nodes are related to the concept: multihop energy sharing path.

v_j. Nodes v_2 and v_3 help relay the energy when node v_1 takes three actions: θ_{12}, θ_{23} and θ_{34}, to transfer its energy to node v_4. The process of energy shared from v_1 to v_4 contains several rounds. We illustrate these rounds in Figure 3. In the figure, there are three rounds listed below.

- The first round contains two actions: θ_{34} and θ_{kj}. θ_{34} transfers a "box" of energy "a" from v_3 to v_4. θ_{kj} transfers a "box" of energy "d" from v_k to v_j.
- The second round contains one action θ_{23}, which transfers a "box" of energy "b" from v_2 to v_3.
- The third round contains one action θ_{12}, which transfers a "box" of energy "c" from v_1 to v_2.

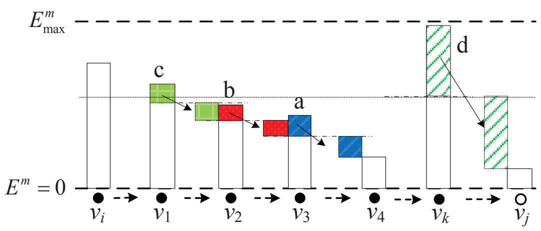

Figure 3 Multi-hop energy sharing rounds and actions

Notice that not all multi-hop energy sharing paths are feasible. For example, suppose that there is a multi-hop energy sharing from v_4 to v_j and v_k is a relay node among them as shown in Figure 3. Because v_k has higher remainder energy than v_4, v_4 cannot transfer any energy to v_j even after v_k firstly transfers a box "d" of energy to v_j. Here, we must especially notice a scenario. For example, there is an other multi-hop energy sharing: $v_i \to v_k \to v_j$ and suppose $E_i^m = 3.5$, $E_k^m = 5$ and $E_j^m = 1$. Although the energy sharing path from v_i to v_j does not seem to be feasible since $E_k^m > E_i^m$, it is actually positive because v_k can transfer two units energy to v_j firstly. From the hints of the example, we can conclude the way to find feasible multihop energy sharing. When the energy consumption on each link, *i.e.*, c, is not included, the way to find the feasible energy sharing paths can be clarified by giving the following lemmas.

Lemma 1: *Suppose that a multi-hop energy sharing from v_i to v_j has h relay nodes and no cost on each link, which are v_l, $l = 1, 2, \cdots, h$. The multi-hop energy sharing is feasible if the following condition is satisfied for any relay node v_q:*

$$\frac{1}{l+1}\left(\sum_{q=0}^{l} E_q^m + E_i^m\right) > \frac{1}{h-l+1}\left(\sum_{q=l+1}^{h} E_q^m + E_j^m\right) \quad (4)$$

for any l.

Proof 1: Please refer to Appendix 7.

According to Lemma 1, the multi-hop energy sharings $v_k \to v_j$ and $v_i \to v_1 \cdots \to v_4$ are feasible and $v_4 \to v_k \to v_j$ is not in Figure 3. Furthermore, when the energy cost on each link is considered, the condition for the feasible energy sharing paths is given in the following lemma.

Lemma 2: *Suppose that a multi-hop energy sharing from v_0 to v_{h+1} has h relay nodes, which are v_l, $l = 1, 2, \cdots, h$. Each link (v_k, v_{k+1}) costs energy $c_{k,k+1}$. The multi-hop energy sharing path is feasible if the following condition is satisfied for any relay node v_q:*

$$\frac{1}{l+1}\left(\sum_{q=0}^{l} E_q^m + E_0^m - \sum_{k=0}^{l} c_{k,k+1}\right) > \frac{1}{h-l+1}\left(\sum_{q=l+1}^{h} E_q^m + E_{h+1}^m\right) \quad (5)$$

for any l.

Proof 2: Please refer to Appendix 7.

3.3 Distributive Actions Selection

It is a straightforward choice to find feasible multi-hop energy sharing as feasible solutions to the problem (1). Because the reward function of each source node is convex and the sum of utility functions is still convex, the above problem is a convex optimization problem. It is easy to find that the constraints in (1) are linear. The problem can be solved in a centralized way by using convex programming techniques such as the Interior Point Method (IPM) [22]. Accordingly, the feasible multi-hop energy sharing paths can be found in such a centralized way. However, it is difficult to solve the problem by IPM in the dynamic case, where the remainder energy of each node is variable during the process of energy sharing because it can harvest energy or it may be affected by the energy dissipation. This dynamics makes the effort complex and difficult to find the feasible paths for multi-hop energy sharing. Suppose that there is a multi-hop energy sharing path from v_i to v_j and v_k is a relay node between them. But v_k receives some energy from other nodes just after the path is established so that Equation (4) cannot be satisfied. At this time, this multi-hop energy sharing path becomes infeasible. Actually, the case can become even more complex when the dynamics of the remainder energy is included. Thus, it is quite complex and costly to look for the multihop energy sharing paths directly.

This paper changes the perspective to solve the problem (1), and gives our solution different from the above straightforward one. Firstly, notice that every action in a multi-hop energy sharing is feasible if the multi-hop energy sharing is feasible according to Lemma 4. Thus, we have the following corollary:

Corrolary 3: *Suppose that there is a set of actions. If every action in this set is feasible, then a multi-hop energy sharing path composed by some actions from this set is also feasible.*

By corollary 3, our scheme for the problem (1) is to find feasible simultaneous actions instead of the multi-hop energy sharing paths in a whole network. For example, the multi-hop energy sharing in the instance of Figure 3 consequently becomes action selection in Figure 4. In this figure, actions are selected simultaneously and may not necessarily belong to a same multi-hop energy sharing path. Because of the hardness of above problem, next section will design an approximate solution to it.

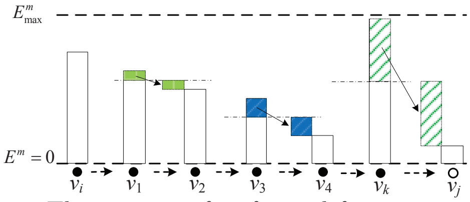

Figure 4 Three actions θ_{12}, θ_{34} and θ_{kj} are simultaneously taken.

4 Design and Analysis of MESS

This section designs and analyzes our scheme MESS for multi-hop energy sharing. In MESS, we propose two algorithms respectively for two cases: static and dynamic energy sharing. In the first case, energy sharing is the only factor to change the remainder energy of each node, such as in night time or the days without sun and the network affords of no other tasks so that no energy is harvested or consumed except shared energy. In the second case, there are other factors to change the remainder energy besides energy sharing, such as energy harvesting or dissipation and consumption caused by network tasks, so the reward obtained by each action may have error. The second one is more realistic but more challenging than the first one.

4.1 Static Energy Sharing

This block presents a Static Energy Sharing Algorithm (SESA), which greedily selects actions with highest rewards. SESA is theoretically proved able to achieve almost optimal reward while its energy sharing cost is also bounded. In this case, each node consumes energy only on energy sharing so the items $E^h_i(\tau-1)$ and $E^c_i(\tau-1)$ in Equation (3) or their sum equal to zero. Equation (3) can be rewriten as follows:

$$E^m_i(\tau) = E^m_i(\tau-1) + E^r_i(\tau-1) - E^t_i(\tau-1), \forall i, \forall \tau \quad (6)$$

By above equation, SESA must find receivers satisfying two conditions in order to maximize the reward of an action θ_{ij}. Firstly, SESA should find nodes with minimal remainder energy $E^m_i(\tau-1)$ in their neighborhoods because these nodes are more eager for energy, *i.e.*, they have higher slope than others according to the reward function in Figure 2. Secondly, these nodes should be able to receive energy as much as possible, *i.e.*, to maximize the item $E^r_j(\tau-1)$ in Equation (6). SESA is given in Algorithm 1. The core idea of SESA is to concurrently select nodes with the minimal remainder energy in their neighborhoods at each time slot τ and these nodes are labeled as receivers, which accordingly forms a receiver set $\mathcal{R}_\tau$ at τ (see Line 3 to 6 in Algorithm 1). In the node set V, the complement of $\mathcal{R}_\tau$ forms the transmitter set $\mathcal{X}_\tau$ (see Line 7 in Algorithm 1). After that, each receiver in $\mathcal{R}_\tau$ finds its transmitter in $\mathcal{X}_\tau$ so that these node pairs forms the action set $\mathcal{G}_\tau$. According to the property of energy charging, a node cannot be a transmitter and receiver simultaneously and a transmitter/receiver can only have one receiver/transmitter at each time slot. By SESA, each receiver finds a transmitter so an action is formed and must contribute positive reward. The action selection process is repeated slot by slot until no action can be selected. See the example shown in Figure 4. Node v_j with the minimal remainder energy is the first selected as a receiver at time slot τ. Because v_4 is not in the maximal range of v_j, v_4 is also selected as a receiver at the same slot. Similarly, v_2 and v_i are selected as transmitters at the same slot. Then, v_j, v_4 and v_2 respectively select v_k, v_3 and v_1 as their transmitters so three actions are selected out at τ. After this, SESA goes to next round, *i.e.*, $\tau+=1$.

Next, we theoretically analyze the properties of MESS. By Algorithm 1, some receivers $v_i \in \mathcal{R}_\tau$ are selected at τ and each v_i of them has its own reward function f_i. Denote the sum of rewards of these nodes by $f_{\mathcal{R}_\tau} = \sum_{v_i \in \mathcal{R}_\tau} f_i$. Let k_τ denote the number of receivers selected at τ, *i.e.*, $k_\tau = |\mathcal{R}_\tau|$, and denote the l^{th} action selected at the time slot by $\theta^l(\tau)$, where $0 < l \leq k_\tau$ and $\theta^l(\tau) \in S_\tau$. So the set of actions is $S^l_\tau = \{\theta^1, \theta^2, \cdots, \theta^l\}$ when the first l^{th} actions are selected till τ. By Algorithm 1, actions are selected one by one. According to the reward function properties shown in Figure 2, nodes with lower remainder energy have bigger slope than others. The following equation consequently holds.

$$\theta^l(\tau) = \arg \max_{\theta^l(\tau) \in \mathcal{X}_\tau \times \mathcal{R}_\tau} \{ \frac{f(S^{l-1}_\tau \cup \{\theta^l(\tau)\}) - f(S^{l-1}_\tau)}{\triangle E(\theta^l(\tau))} \} \quad (7)$$

where $\triangle E(\theta^l(\tau))$ denotes the received energy through the action $\theta^l(\tau)$, and $\theta^l(\tau) > 0$ is obtained through the step 9 of Algorithm 1. Equation (7) means that the Algorithm 1 always selects the actions which can achieve the maximal reward per unit energy. We then can have the following lemma:

Algorithm 1 Static Energy Sharing Algorithm (SESA).

Input: The node set V and the reward function f_i for each node v_i;
Output: A sequence $\mathcal{G}_T$ of actions in the period T.

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for Each round τ ∈ T do
    Let X_τ = V and R_τ = ∅;
    while X_τ ≠ ∅ do
        Select a node v_j with minimal remainder energy
        in X_τ, and add them into R_τ;
        X_τ = X_τ/N_j, where v_j ∈ N_j;
    end while
    X_τ = V/R_τ ;
    for Each node v_j ∈ R_τ in parallel do
        v_j selects a reciever v_i in X_τ ∩ N_j such that v_i =
        arg max_{v_i∈X_τ} {E_i(τ) − E_j(τ) − c_ij}, i.e., an action
        θ_ij(τ) is selected;
        v_i transfers (E_i^m(τ−1) − E_j^m(τ−1) − c_ij)/2 energy to v_j;
        Add the action θ_ij(τ) into the set S_τ;
        Delete v_j from R_τ;
    end for
    G_τ = G_τ ∪ {S_τ};
    τ+ = 1;
end for
Output the a sequence G_T of actions.
```

Lemma 4: *For an arbitrary set S of actions, its overall reward is bounded by the following inequality.*

$$f(S) \leq f(S_\tau) + \triangle E(S)\{\frac{f(S_\tau \cup \{\theta^m(\tau)\}) - f(S_\tau)}{\triangle E(\theta^m(\tau))}\} \quad (8)$$

where θ^m denotes the action able to achieve maximal reward per unit energy in S.

Proof 3: Please refer to Appendix 7.

Notice that the set S is arbitrary in above lemma and it can be assumed to be obtained by the optimal solution for the problem (1). Although the optimal solution cannot be obtained directly, Lemma 4 gives us hints that the reward obtained by Algorithm 1 can be very close to the optimal one, which is identified by the following theorem.

Theorem 5: *The overall network reward obtained by Algorithm 1 can achieve $1 - \frac{1}{e}$ approximation for the problem (1).*

Proof 4: Let $\mathcal{G}^*$ denote the optimal scheme to problem (1) and also the action sets obtained by the optimal energy sharing scheme. The maximal reward obtained by this optimal solution is denoted by $f(\mathcal{G}^*)$. Notice that a period consists of some time slots, and at each time slot there are τ_k^* nodes selected by the optimal scheme. Let $K = \sum_{\tau\in T} k_\tau$. So we have the subset $\mathcal{G}^* = \bigcup_{\tau\in T} S_\tau^*$, where S_τ^* is the set of actions selected by optimal scheme at τ. Let $\Delta_{\tau_l} = f(S_\tau^*) - f(S_\tau^l)$. So $f(S_\tau^*) \leq f(S_\tau) + \triangle E(S_\tau^*)\frac{f(S_\tau \cup \{\theta^m(\tau)\}) - f(S_\tau)}{\triangle E(\theta^m(\tau))}$ according to Lemma 4, where $\theta^m(\tau)$ is the action able to achieve maximal reward per unit energy in S_τ, *i.e.*, $\triangle_\tau \leq \triangle E(S_\tau^*)\frac{f(S_\tau^l \cup \{\theta^m(\tau)\}) - f(S_\tau^l)}{\triangle E(\theta^m(\tau))}$. According to Algorithm 1, the action able to achieve maximal reward is selected at each time by Equation (7). Thus, $\triangle_{\tau_l} \leq \triangle E(S_\tau^*)\frac{\triangle_{\tau_l} - \triangle_{\tau_{l+1}}}{\triangle E(\theta^m(\tau))}$. By transforming this inequality, we can have $\triangle_{\tau_{l+1}} \leq \triangle_{\tau_l}(1 - \frac{\triangle E(\theta^m(\tau))}{\triangle E(S_\tau^*)})$. Unrolling this inequality, we can get $\triangle_{k_\tau} \leq \triangle_{\tau_1}\prod_{l=1}^{k_\tau}(1 - \frac{\triangle E(\theta^m(\tau))}{\triangle E(S_\tau^*)})$. Notice that $\triangle E(\theta^m(\tau))$ is the one obtaining maximal value in S_τ^* so it must be larger than the average of those in this set, *i.e.*, $\triangle E(\theta^m(\tau)) \geq \frac{1}{k_\tau}\triangle E(S_\tau^*)$. We can have $\triangle_{k_\tau} \leq \triangle_{\tau_1}\prod_{l=1}^{k_\tau}(1 - \frac{1}{k_\tau}) = \triangle_{\tau_1}(1 - \frac{1}{k_\tau})^{k_\tau} < \triangle_{\tau_1}\frac{1}{e} < f(S_\tau^*)\frac{1}{e}$ because $\triangle_{\tau_1} = f(S_\tau^*) - f(S_\tau^1) < f(S_\tau^*)$. Because $\triangle_{k_\tau} = f(S_\tau^*) - f(S_\tau)$, $f(S_\tau > (1 - \frac{1}{e})f(S_\tau^*)$. Unrolling this inequality slot by slot, we have $\sum_{\tau\in T} f(S_\tau) > \sum_{\tau\in T}(1 - \frac{1}{e})f(S_\tau^*)$, *i.e.*, $f(\mathcal{G}_T) > (1 - \frac{1}{e})f(\mathcal{G}_T^*)$. This finishes proof.

Another object of this paper is to bound the overall cost of energy sharing by Algorithm 1. Let $\mathcal{G}$ denote our scheme and action set obtained by the scheme. Let $c(\mathcal{G}^*)$ denote the energy cost by $\mathcal{G}^*$ and $c(\mathcal{G})$ denote that by MESS. Suppose that there is another scheme $\mathcal{G}'$, which always takes each action by finding nearest neighbor for each receiver. All these actions by $\mathcal{G}'$ form a scheme set $\mathcal{G}'$. $c(\mathcal{G}')$, $c(\mathcal{G}^*)$ and $c(\mathcal{G})$ are the sum of the energy cost of all actions respectively in $\mathcal{G}'$, $\mathcal{G}^*$ and $\mathcal{G}$. Obliviously, there must be $c(\mathcal{G}') \leq c(\mathcal{G}^*)$. Let $\mathcal{X}'$ denote the set of source nodes obtained by $\mathcal{G}'$. We can have the following lemma to bound the maximal energy cost by our scheme.

Theorem 6: *The maximal energy cost of the scheme MESS is bounded by the optimal one with $c(\mathcal{G}) \leq (n-1)\max\{c(\mathcal{G}^*), E_{max}^m\}$.*

Proof 5: See the Appendix 7.

4.2 Dynamic Energy Sharing

The practical scenario during energy sharing is that the remainder energy of each node may be variable, which is caused by following several reasons. The first is that each node is equipped with energy harvesting devices, such as a solar penal, and able to harvest natural energy source. The second is that a node may afford of tasks other than energy sharing, such as communication. The third is energy dissipation caused by energy sharing. We combine the energy variation caused by the above reasons into one item $\delta E_i(\tau)$ at τ and have the following equation based on Equation (3) and (6).

$$\delta E_i(\tau) = E_i^h(\tau - 1) - E_i^c(\tau - 1), \forall v_i, \forall\tau \quad (9)$$

Since the capacity of each node's batteries is limited and the above energy variation is relative small according to the survey of Section 6 and the assumption in Section 2, there must be $-E_i^m(\tau-1) < \delta E_i(\tau) \leq E_{max}^m$ and $\frac{\delta E_i(\tau)}{E_{max}^m} \ll 1$. A quite practical problem is how the energy variation affects the overall reward of a network. We give theoretical analysis and conclude our it into Theorem 7.

When each transmitter or receiver has energy variation, the reward achieved by each action $\theta_{ij}(\tau)$ correspondingly has error $\epsilon_{ij}(\tau)$, *i.e.*, $f(\theta_{ij}(\tau)) + \epsilon_{ij}(\tau)$. The real value of $\epsilon_{ij}(\tau)$ is dynamic and determined by $\delta E_i(\tau)$ and composes of two parts of energy variation from transmitter and receiver. But it is difficult to model the distribution of the energy variation of each node because this distribution is quite application-dependent. In this section, our solution need not know the energy variation distribution. This section gives a simple way to analyze the impact of energy variation on the network reward. Suppose it takes time $\delta\tau$ to finish energy sharing after a target node v_j finds its source node v_i at the beginning of time slot τ, *i.e.*, an action $\theta_{ij}(\tau)$ is selected. Both source and target nodes may have their energy variation $\delta E_i(\tau)$ or $\delta E_j(\tau)$ during $\delta\tau$. It is easy to find that the action $\theta_{ij}(\tau)$ can contribute more reward than that expected at the beginning of τ when $\delta E_i(\tau) > 0$ or $\delta E_j(\tau) < 0$. Otherwise, the action can contribute less reward. The energy variation of both v_i and v_j at current time slot τ cannot be known at the beginning of this slot. In our solution, the energy variation at precious time slot $\tau - 1$ is applied to estimate the energy variation at τ, where each transmitter or receiver has its energy variation with probability $p_i(\tau - 1) = \frac{E_{max}^m + \delta E_i(\tau-1)}{2E_{max}^m}$ or $p_j(\tau-1) = \frac{E_{max}^m + \delta E_j(\tau-1)}{2E_{max}^m}$. Let $p_{ij}(\tau-1) = p_i(\tau-1)p_j(\tau-1)$. When transmitter or receiver has higher energy variation, the action can achieve more reward with probability. Notice that each receiver must correspond to a transmitter so $\sum\limits_{v_i \in \widetilde{\mathcal{X}}_\tau \cap \mathcal{N}_j / \widetilde{\mathcal{R}}_\tau} p_{ij}(\tau-1) = 1$ for each receiver v_i. In order to select some actions to maximize the overall reward in the case of dynamic energy sharing, this section designs an algorithm, called DESA, as given in Algorithm 2, when the reward contributed by each action is dynamics in the process of energy sharing.

By Algorithm 2, the total reword of a network is bounded in the worst case when energy variation exists. By the scheme$\widetilde{\mathcal{G}}_T$ in Algorithm 2, some actions are selected out at each time slot and denoted by $\widetilde{\theta}(\tau)$. The energy variation of an action $\delta\theta_{ij}(\tau)$ is denoted by $\delta\theta_{ij}(\tau) = \delta E_i(\tau) - \delta E_j(\tau)$, where energy is transferred from v_i to v_j. The reward variation of this action caused by its energy variation is $f(\delta\theta_{ij}(\tau))$. For convenience, we denote $\theta_{ij}(\tau)$ by $\theta^l(\tau)$, which indicates that $\theta^l(\tau)$ is the l^{th} action selected at τ. Thus, Algorithm 2 actually selects each action able to achieve maximal reward when the energy variation exists with probability, *i.e.*, the action $\theta^l(\tau)$ satisfies following equation:

Algorithm 2 Dynamic Energy Sharing Algorithm (DESA)

Input: The node set V and the reward function f_i for each node v_i.
Output: Energy sharing Scheme $\widetilde{\mathcal{G}}_T$ for all source nodes $v_i \in \widetilde{\mathcal{X}}_T$ in the period T;

```
for Each round τ ∈ T do
    Let X̃_τ = V and R̃_τ = ∅;
    while X̃_τ ≠ ∅ do
        Select a node v_j with minimal remainder energy in X̃_τ, and add them into R̃_τ;
        X̃_τ = X̃_τ / N_j, where v_j ∈ N_j;
    end while
    X̃_τ = V / R̃_τ;
    for Each node v_j ∈ R̃_τ in parallel do
        v_j calculates the probability p_ij(τ−1), and selects a receiver ṽ_i with highest probability p_ij(τ), in X̃_τ ∩ N_j such that v_i = arg max_{v_i∈X_τ} {E_i(τ) − E_j(τ) − c_ij}, i.e., an action θ̃_ij(τ) is selected;
        v_i transfers (E_i^m(τ−1) − E_j^m(τ−1) − c_ij)/2 energy to v_j;
        Add the action θ̃_ij(τ) into the set S̃_τ;
        Delete v_j from R̃_τ;
    end for
    G̃_τ = G̃_τ ∪ {S_τ};
    τ+ = 1;
end for
Output the energy sharing Scheme G̃_T.
```

$$\theta^l(\tau) = \arg \max_{\theta^l(\tau) \in \mathcal{X}_\tau \times \mathcal{R}_\tau} \{ \frac{f(S_\tau^{l-1} \cup \{\theta^l(\tau)\}) - f(S_\tau^{l-1})}{\triangle E(\theta^l(\tau))} + p^l(\tau-1) f(\delta\theta^l(\tau)) \}$$

Above equation states a fact that energy variation at current time slot cannot be known at the beginning of this slot and may be only estimated by that at previous slot but the actual value of reward error can be precisely calculated afterwards. It is the exploitation or exploration problem but we can catch the reward error theoretically. In Algorithm 2, the action with highest probability is selected and each action may create error on its reward. Denote this reward error created by the action by $\epsilon^l(\tau)$, *i.e.*, $\epsilon^l(\tau) = f(\delta\theta^l(\tau))$. On the reward errors of all actions selected by Algorithm 2, this section has the following theorem.

Theorem 7: *When energy variation exists during energy sharing, the overall reward obtained by Algorithm 2 is not less than* $(1 - \frac{1}{e}) f(\mathcal{G}^*) + \sum_{\tau \in \mathrm{T}} E(\triangle_\tau)$ *with high probability, where* $\mathcal{G}^*$ *is the action set of the optimal scheme,* $\mathbb{E}(\triangle_\tau) = \delta_\tau \sum\limits_{l=1}^{k_\tau} \triangle_{\tau_l}$ *and* $\Delta_{\tau_l} = f(S_\tau^*) - f(S_\tau^l)$.

Proof 6: According to Lemma 4, we can easily obtain the following equation:

$$f(S) \leq f(\widetilde{S}_\tau) + \triangle E(S)\{\frac{f(\widetilde{S}_\tau \cup \{\widetilde{\theta}^m(\tau)\}) - f(S_\tau)}{\triangle E(\widetilde{\theta}^m(\tau))}\} \quad (10)$$

Let $\mathcal{G}^*$ be an optimal scheme to problem (1) and the maximal reward obtained by this optimal solution is denoted by $f(\mathcal{G}^*)$, where $\mathcal{G}^*$ is the optimal energy sharing schedule set by the optimal. The proof of this theorem is quite similar to that of Theorem 5. Let $\Delta_{\tau_l} = f(S_\tau^*) - f(\widetilde{S}_\tau^l)$. So $f(S_\tau^*) \leq f(\widetilde{S}_\tau) + \triangle E(S_\tau^*)\frac{f(\widetilde{S}_\tau \cup \{\widetilde{\theta}^m(\tau)\}) - f(\widetilde{S}_\tau)}{\triangle E(\widetilde{\theta}^m(\tau))}$ according to Equation (10), where $\widetilde{\theta}^m(\tau)$ is the action able to achieve maximal reward per unit energy in $\widetilde{S}_\tau$, *i.e.*, $\triangle_\tau \leq \triangle E(S_\tau^*)\frac{f(\widetilde{S}_\tau^l \cup \{\widetilde{\theta}^m(\tau)\}) - f(\widetilde{S}_\tau^l)}{\triangle E(\widetilde{\theta}^m(\tau))}$. So $\triangle_{\tau_l} \leq \triangle E(S_\tau^*)\frac{\triangle_{\tau_l} - \triangle_{\tau_{l+1}}}{\triangle E(\widetilde{\theta}^m(\tau))}$. By transforming this inequality, we can have $\triangle_{\tau_{l+1}} \leq \triangle_{\tau_l}(1 - \frac{\triangle E(\widetilde{\theta}^m(\tau))}{\triangle E(S_\tau^*)})$. Recall that each actions in $\widetilde{S}_\tau$ obtained by Algorithm 2 may create error $\epsilon\widetilde{\theta}^m(\tau)$ with probability $p^l(\tau - 1)$ because $\triangle E(\widetilde{\theta}^m(\tau))$ is estimated at previous time slot $\tau - 1$ in Algorithm 2, *i.e.*, $\triangle E(\theta^m(\tau)) = \triangle E(\widetilde{\theta}^m(\tau)) + p^m(\tau)\delta\theta^m(\tau)$. So we have $\triangle_{\tau_{l+1}} \leq \triangle_{\tau_l}(1 - \frac{\triangle E(\theta^m(\tau))}{\triangle E(S_\tau^*)} + \frac{p^m(\tau)\delta\theta^m(\tau)}{\triangle E(S_\tau^*)}) = \triangle_{\tau_l}(1 - \frac{\triangle E(\theta^m(\tau))}{\triangle E(S_\tau^*)}) + \triangle_{\tau_l}\frac{p^m(\tau)\delta\theta^m(\tau)}{\triangle E(S_\tau^*)}$. Let $\sigma_\tau^m = \frac{p^m(\tau)\delta\theta^m(\tau)}{\triangle E(S_\tau^*)}$ so we have the following inequality:

$$\triangle_{\tau_{l+1}} \leq \triangle_{\tau_l}(1 - \frac{\triangle E(\theta^m(\tau))}{\triangle E(S_\tau^*)}) + \sigma_\tau^m \triangle_{\tau_l} \quad (11)$$

Notice that there must be $0 < \frac{\triangle E(\theta^m(\tau))}{\triangle E(S_\tau^*)} < 1$ and thus $0 < 1 - \frac{\triangle E(\theta^m(\tau))}{\triangle E(S_\tau^*)} < 1$. So unrolling the inequality (11), we can get $\triangle_{k_\tau} \leq \triangle_{\tau_1}\prod_{l=1}^{k_\tau}(1 - \frac{\triangle E(\widetilde{\theta}^m(\tau))}{\triangle E(S_\tau^*)})$.

$$\triangle_{k_\tau} \leq \triangle_{\tau_1}\prod_{l=1}^{k_\tau}(1 - \frac{\triangle E(\theta^m(\tau))}{\triangle E(S_\tau^*)}) + \sum_{l=1}^{k_\tau}\sigma_\tau^l \triangle_{\tau_l} \quad (12)$$

Similar to the proof of Theorem 5, we can have $f(S_\tau) > (1 - \frac{1}{e})f(S_\tau^*) - \sum_{l=1}^{k_\tau}\sigma_\tau^l\triangle_{\tau_l}$. Unrolling this inequality time slot by time slot, we have $\sum_{\tau\in T} f(S_\tau) > \sum_{\tau\in T}(1 - \frac{1}{e})f(S_\tau^*) - \sum_{\tau\in T}\sum_{l=1}^{k_\tau}\sigma_\tau^l\triangle_{\tau_l}$. $f(\mathcal{G}_T) > (1 - \frac{1}{e})f(\mathcal{G}_T^*) - \sum_{\tau\in T}\mathbb{E}(\triangle_\tau)$, where $\mathbb{E}(\triangle_\tau) = \sum_{l=1}^{k_\tau}\sigma_\tau^l\triangle_{\tau_l}$. This finishes proof.

Next, this section considers a special case of above theorem, where the error happens uniformly and randomly on each action. Thus, the reward error is given in the following lemma according to Theorem 7.

Theorem 8: *When energy variation exists during energy sharing and happens uniformly and randomly on each action, the overall reward obtained by Algorithm 2 satisfies the following inequality:*

$$f(\widetilde{\mathcal{G}}_\tau) > (1 - \frac{2}{e})f(\mathcal{G}_\tau^*) \quad (13)$$

The proof of this theorem is quite straightforward. Notice that the error item $\sum_{l=1}^{k_\tau}\sigma_\tau^l\triangle_{\tau_l}$ in Theorem 7. $\sum_{l=1}^{k_\tau}\sigma_\tau^l\triangle_{\tau_l} = \sigma_\tau\sum_{l=1}^{k_\tau}\triangle_{\tau_l} \leq \sigma_\tau\triangle_{\tau_1}[(1 - \frac{1}{2}) + (1 - \frac{1}{3})^2 + \cdots + (1 - \frac{1}{k_\tau})^{k_\tau - 1}] < \sigma_\tau\frac{k_\tau}{e}f(S_\tau^*) < \frac{1}{e}f(S_\tau^*)$.

5 Evaluation

This paper chooses simulation as the primary tool for investigation in order to understand system behaviors at scale. The simulation for the performance evaluation of $MESS$ is conducted by the Omnet++ simulation tool [17]. In this simulation, each node is set to have some initial energy of $1mAH$ (*i.e.*, $3600mAs$) and the energy it can harvest energy at each time slot is random variable with expectation $15mA$ and variance $15mA$. Nodes are deployed randomly in a size-fixed area with 500×500 square meters. Each node samples data and creates a packet per minute. The energy sharing cost is set as $0.6(\frac{d}{d_{max}})^2\Delta E$, where d_{max} is the maximal distance a node can transfer its energy with all of its initial energy. In our simulation, MAC layer and network layer respectively implement the IEEE 802.11 protocol [23] and the minimum hop-count routing. Thus, this paper directly uses the DESA algorithm because the protocol and routing consumes much energy on communication in the process of energy sharing.

The simulation results are shown in Figure 5, 6, 7 and 8, which compare the performance of the network working respectively by the energy sharing algorithm DESA and no energy sharing algorithm. Figure 5 shows the network working by the energy sharing algorithm DESA has low average remainder energy than that by no energy sharing algorithm. This is caused by the energy consumption on energy sharing and message communications for it as shown in Figure 8. Figure 8 shows the average energy cost under different number of nodes.

Figure 5 does not mean a network working under DESA has shorter life time than that under no energy sharing. Figure 7 shows the variance of remainder energy, which indicates the average square of those differences between the average remainder energy and the remainder energy of each node. We pick out 10 nodes from the simulation, in which there are 150 nodes totally. The IDs of these 10 nodes range from 0 to 9. Because all nodes were randomly deployed, these 10 nodes are also picked out randomly. As shown in Figure 6, these 10 nodes have different remainder energy respectively when MESS and no energy sharing algorithm are respectively taken. When no energy sharing algorithm is adopted, nodes v_0, v_4 and v_9 remain energy of 22.89, 92.832 and 63.447 mAs respectively, which are much lower than their remainder energy by using MESS. These nodes will first die out so the network must stop working. Meanwhile, the node

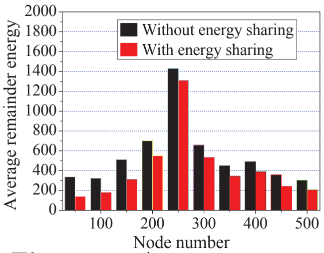

Figure 5 Average remainder energy (Unit:mAs)

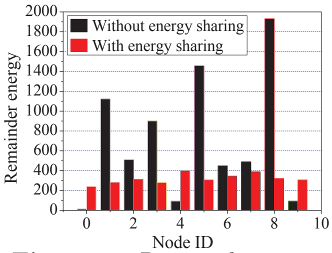

Figure 6 Remainder energy of several nodes (Unit:mAs)

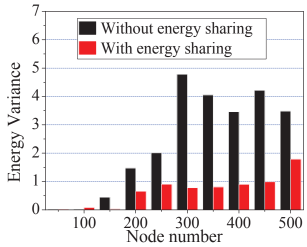

Figure 7 Energy variance (Unit:mAs)

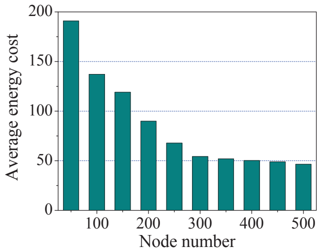

Figure 8 Energy cost (Unit:mAs)

with minimal remainder energy is v_0 by the algorithm DESA, which has 240.29 mAs remained.

The main factor to affect the cost of energy sharing is the distance among nodes. Because nodes were deployed in a size-fixed area, the average distance among nodes is lower when the total number of nodes is smaller as shown in Figure 8. We also find that higher node density will not necessarily lead to lower energy cost during energy sharing. For example, the energy costs are no big difference when the number of nodes are from 300 to 500 in Figure 8. When the node density is too high, wireless channel competition is accordingly high. Some messages for energy sharing are lost so some actions cannot be finished. That is also the reason why the energy variance is higher when the number of nodes increases in Figure 7.

6 Related Work

Energy sharing by wireless energy transfer has been widely used in some areas, such as RFID systems with passive tags [14][24]. Kurs *et al.* experimentally demonstrated efficient nonradiative power transfer over distance up to 8 times the radius of the coils by using self-resonant coils in a strongly coupled regime after the effort of Nikola Tesla on wireless energy transfer in the early 20th century [18]. The newly discussed technique: wireless charging, promoted the development of its applications in wireless networks. There have been increasingly works [7][8][9][10][2] focusing on energy sharing in multi-hop sensor networks but they did not involve the multi-hop energy sharing. Zhu *et al.* introduced energy sharing into sensor networks and made an interesting and feasible attempt in energy router and the related protocol designing [7]. Tong *et al.* investigated the impact of wireless charging technology on sensor network deployment and routing arrangement and developed heuristic algorithms to solve their formalized deployment and routing problem [10]. There are other works arguing to charge senor nodes with mobile chargers [9][2][25]. They also attempted to find optimal traveling paths for mobile charging vehicles. In the existing works on energy sharing in sensor networks, energy is transferred from a base station to sensor nodes by mobile chargers. This paper is quite different from previous work and researches multi-hop energy sharing and high performce scheme for it.

Energy harvesting devices or platforms, such as Helimote [26], Prometheus [27] and AmbiMax [1], were preciously attempted to provide the sustainable operation for WSNs. But it may worsen the energy fairness among nodes because the amount of energy each node can harvest is limited and random [28][29]. The ambient energy, *e.g.*, solar, wind, is often not intensive enough to sustain the continuous full duty cycle for sensor nodes in long term operation [30][27][31]. Gu *et al.* established experiment and their results showed that the duty cycles of an energy-harvesting node can only range from 0.2% to 9.78% [28]. The experiments in [32] showed that the energy harvested by a solar panel in each day is less than 10% of an AA type NiMH battery with the capacity of 2200mAH. Although the harvest energy in each day is quite small comparing to the batteries' capacity, it will accumulate or may be wasted when it is not used in 20 or more days. This paper considers the affection of some factors including energy harvesting and consumption on calculation and so on during energy sharing. It is not considered by precious related works.

7 Conclusion

This paper proposed a novel energy sharing scheme: multi-hop energy sharing, based on the newly researched energy sharing technique. To our best knowledge, we are the first to propose this kind of energy sharing schemes. Different from data communication, it is quite challenging to find feasible energy sharing paths. We give the condition to find feasible multi-hop energy sharing paths. Based on the condition, we designed the scheme $MESS$ to find the best feasible multi-hop energy sharing paths so as to maximize the overall reward. In $MESS$, two algorithms, SESA and DESA, were respectively designed for two cases: static and dynamic energy sharing. The theoretical performance of $MESS$ for both algorithms is almost $1-1/e$ of the optimal solution. Our simulation was carefully designed and implemented, and the experimental results showed that energy fairness can be improved evidently. The scheme of this paper is centralized, and we take it as a future work to design a localized one.

Acknowledgement

This work is under the support of the General Program of the National Natural Science Foundation of China (NSFC) under Grants No. 61473109 and 61003298, the Zhejiang Province Natural Science Foundation under Grant No.LY14F020042 and LY14F020047, and the Graduate Scientific Research Foundation of Hangzhou Dianzi University.

Appendix

The proof for Lemma 1.

Proof 7: We prove the lemma by induction.
Step (1): When $h = 0$, there is no relay node between the source and the target nodes. Equation (4) becomes:

$$E_i^m > E_j^m \tag{14}$$

It is a necessary condition because v_i transfers its energy to v_j and must have more energy than later.
Step (2): When $h = 1$, there is one relay node. There are two cases: $l = 0$ and $l = 1$ according to Equation (4). In the case $l = 0$, Equation (4) becomes:

$$E_i^m > \frac{1}{2}(E_1^m + E_j^m) \tag{15}$$

In the case $l = 1$, Equation (4) becomes:

$$\frac{1}{2}(E_i^m + E_1^m) > E_j^m \tag{16}$$

When Equation (15) and (16) are satisfied simultaneously, we discuss in two case: $E_1^m > E_j^m$ and $E_1^m < E_j^m$ because the step (2) is equivalent to the step (1) when $E_1^m = E_j^m$. When $E_1^m > E_j^m$, v_1 can transfer $\frac{1}{2}(E_1^m - E_j^m)$ to v_j. So the remainder energy of v_1 and v_j are updated to $\frac{1}{2}(E_1^m + E_j^m)$. According to Equation (15), v_i can still transfer its energy to v_1 and v_j. When $E_1^m < E_j^m$, v_i can transfer $\frac{1}{2}(E_i^m - E_1^m)$ to v_1. So the remainder energy of v_i and v_1 are updated to $\frac{1}{2}(E_i^m + E_1^m)$. According to Equation (16), v_i and v_1 can still transfer their energy to v_j.
Step(3) Suppose Lemma 1 is correct when there are more than one relay node, *i.e.*, v_i can transfer its energy to v_j when there are h ($h > 1$) relay nodes among them. Without loss of generality, suppose that relay node v_l ($l = 1, 2, \cdots, h$) is closer to v_j than v_{l-1}. In other words, these h nodes are arranged into an increasing order from v_i to v_j. Now we prove that Lemma 1 is also correct when there are $h + 1$ relay nodes. Let the $h + 1^{th}$ node be v_{h+1}, which can locate in any place among the previous $h + 2$ nodes. Suppose that v_{h+1} locates between v_l and v_{l+1}. Thus, there is a multihop energy sharing path: $v_i \to v_1 \cdots \to v_l \to v_{h+1} \to v_{l+1} \to \cdots \to v_h$. Since we suppose Lemma 1 is correct when there are h relay nodes, the two subblocks: $v_i \to v_1 \cdots \to v_l$ and $v_{l+1} \to \cdots \to v_h$, are feasible. If we treat these two subblocks as two "nodes", then it seems there are two big node and a relay node v_{h+1}. Thus, the proof is similar to that in Step (2).

This finishes the proof of this lemma.

The proof for Lemma 2.

Proof 8: Let $IE_l^m = E_l^m - c_{l,l+1}$, and then Equation (5) becomes:

$$\frac{1}{l+1}(\sum_{q=0}^{l} IE_q^m + IE_0^m) > \frac{1}{h-l+1}(\sum_{q=l+1}^{h} E_q^m + E_{h+1}^m)$$

Base on above equation, the proof of Lemma 2 is similar to that for Lemma 1.

The proof for Lemma 4.

Proof 9: We assume that there is an arbitrary set S of actions, in which each action achieves nonnegative reward. Let $S = \{\theta^1, \theta^2, \cdots \theta^{k_\tau}\}$ and S_τ is an actions set selected by Algorithm 1, which achieve nonnegative reward. When the $l^{th} \in S$ action is taken, we can obtain the additional reward, denoted by $\triangle f^l$, and $\triangle f^l = f(S_\tau \cup \{\theta^1, \theta^2, \cdots, \theta^l\}) - f(S_\tau \cup \{\theta^1, \theta^2, \cdots, \theta^{l-1}\})$. Let $E^m (\in S)$ be an action satisfying: $\theta^m = \arg\max_{\theta^l \in S} \frac{f(S_\tau \cup \{\theta^l(\tau)\}) - f(S_\tau)}{\triangle E(\theta^l)}$. Because the reward function is convex, non-decreasing, we can obtain the following result:

$$\begin{aligned} f(S_\tau \cup S) &= f(S_\tau) + \sum_{l=1}^{k_\tau} \triangle f^l \\ &\le f(S_\tau) + \sum_{l=1}^{k_\tau} (f(S_\tau \cup \{\theta^l(\tau)\}) - f(S_\tau)) \\ &\le f(S_\tau) + \sum_{l=1}^{k_\tau} \triangle E(\theta^l(\tau)) \{\frac{f(S_\tau \cup \{\theta^m(\tau)\}) - f(S_\tau)}{\triangle E(\theta^m)}\} \\ &= f(S_\tau) + \triangle E(S) \frac{f(S_\tau \cup \{\theta^m\}) - f(S_\tau)}{\triangle E(\theta^m)} \end{aligned}$$

where $\triangle E(S)$ equals to the sum of the received energy through the actions in S. Thus, we can obtain the following equation for any such S, $S_\tau \in \mathcal{G}$.

$$\frac{f(S \cup S_\tau) - f(S_\tau)}{\triangle E(S)} \le \max_{\theta^l(\tau) \in S_\tau} \frac{f(S_\tau \cup \{\theta^l(\tau)\}) - f(S_\tau)}{\triangle E(\theta^l(\tau))}$$

This equation indicates that the reward f per unit energy achieved by selecting a sequence of energy sharing actions S_2 is always bounded by the maximum reward per unit energy achieved by one action, over all energy sharing actions $\theta^m(\theta)$ among this sequence of actions.

Furthermore, we can have $f(S) \le f(S \cup S_1)$ by monotonicity so $f(S) \le f(S_\tau) + \triangle E(S) \frac{f(S_\tau \cup \{\theta^m\}) - f(S_\tau)}{\triangle E(\theta^m)}$.

The proof for Lemma 6.

Proof 10: We denote the action set $\theta_{ij}(\tau)$ selected by an arbitrary schemex. There are two cases: $\theta_{ij}(\tau) \in \mathcal{X}'$ and $\theta_{ij}(\tau) \notin \mathcal{X}'$. If the distance d_{ij} is the minimal one among all neighbors of the target node v_j, then there must be $\theta_{ij}(\tau) \in \mathcal{X}'$ and the energy cost of this action $c(\theta_{ij}(\tau))$ is minimal. If the nearest neighbor of v_j is v_m, where $m \neq i$, then $d_{ij} > d_{mj}$ and $c(\theta_{ij}(\tau)) > c(\theta_{mj}(\tau))$. There is a multi-hop energy sharing S'_τ containing some energy sharing actions, $S'_\tau = \{\theta_{il}, \cdots, \theta_{km}, \theta_{mj}\}$, in the original network. Among all these actions in S'_τ, all the target nodes select their nearest neighbors as source nodes. The first source node in S'_τ is v_i and same with that of d_{ij} and the last target node in S'_τ is v_j and same with that of d_{ij}. Notice that d_{mj} belongs to $\mathcal{X}'$ and other actions in this multi-hop energy sharing must not belongs to $\mathcal{X}'$. Thus, there must be $d_{ij} \leq \theta_{il} + \cdots + \theta_{km} + \theta_{mj} = \Sigma_{\theta_{km} \in S'_\tau} d_{km}$. Recall that there are totally n so S'_τ can contain at most $n-1$ actions. Therefore, there must be $d_{ij} \leq \Sigma_{\theta_{km} \in S'_\tau} d_{km} \leq (n-1)d_{mj}$, where d_{mj} denotes the maximal one among S'_τ. Recall that the energy cost model is $c_{ij} = \alpha d_{ij}^\beta$ so $c(\theta_{ij}(\tau)) \leq \Sigma_{\theta_{km} \in S'_\tau} \alpha d_{km}^\beta \leq (n-1)\alpha d_{mj}^\beta = (n-1)c(\theta_{mj}(\tau))$. Because the battery capacity of each node is limited with E_{max}^m, there must be $c(\theta_{ij}(\tau)) \leq \max\{(n-1)c(\theta_{mj}(\tau)), E_{max}^m\}$. Unrolling all actions in all time slots, we then have $c(\mathcal{G}) \leq (n-1)\max\{c(\mathcal{G}'), E_{max}^m\}$. Then, $c(\mathcal{G}) \leq (n-1)c(\mathcal{G}') \leq (n-1)\max\{c(\mathcal{G}^*), E_{max}^m\}$.

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